INTORDUCTION TO SET THEORY FOR BEGINNERS

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ABSTRACT

Mathematics as a wide field of study engulfs a topic called ~Set Theory among other numerous topics' It is a system of summarising or analysing an information for an organization. It is used in a similar way as statistics with some notations that are peculiar to it. It differentiates between finite and infinite sets and also establishes collection properties of sets in terms of their similarities and differences. It entails diagrams which are used to solve several Mathematical problems. The introductory stage of set theory explains the collection Items, Objects and Things because of certain similarities or differences which they possess. They have peculiar notations such as Alpha, Betta, Omega, Landa, Phile and capital letters like A, B, C, etc. Which are as related to Human daily activities.*

Introduction

A set is a collection of items, objects or things together because of certain similarities which they possess. Members of particular set always have something in common, such as:

1. Set of students of languages, among which we have set of male students and set of female students.
2. A set of mathematical instrument in which we have compasses, ruler, protractor, dividers, 45^o set square, 60^o set square, eraser and sharpener. M and could be used to represent mathematical instrument. Members of this set M have one similarity. They are used for drawing and measuring shapes.
3. A set of chair, table, desk, bench and stood. Members of this set are made of wood. So, we could represent wood and therefore W= (chair, table, stood, desk, bench)

However, there may be no obvious similarity between the members of a set, that is why it is called collection a well-defined objects or things e.g. members of H are Goat, Book. Road, Iron.

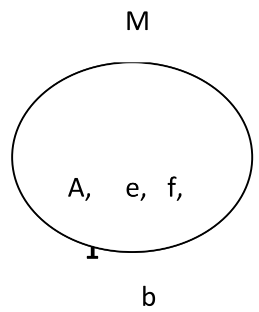
H = (Goat, Book, Road, Iron)

Members of set H above don't seem to have anything in common. Members of a set also referred to as elements of that set.

X= (pot, 5, car)

Capital letters are used to represent sets, while small letters are used to represent their elements e.g. M = (b, e, f, a) set are identified using curly brackets i.e. { }

Or Venn diagram i.e



A set can be described, or defined, in two ways: either by making a list of its members, or by describing the property or rule that connects it's members or showing the members within a boundary i.e. Venn diagram.

Examples:

1. Listing of elements

(a) W = (Ilorin, Ibadan, Mina, Lokoja)

(b) X = (Tree, Field, Pole, Grass)

2. Description by Property or Rules Connecting the Elements

(a) Y = (School Prefects

(b) β = (Even Numbers)

(c) $\propto$ = (x: 5 < x< 17)

(d) G = (Prime numbers between 10 and 30)

If a set is specified by listing its elements, we call it <u>tabular form</u> of a set, and if it is specified by stating its properties, such as F = (x:x is odd), then it is called the set <u>builder form.</u>

Empty Sets

Empty set means a set which contains nothing. It represented as { }. $\emptyset$ or referred to as null set e.g { CAILS students of age 10 years} { Igbo people who are indigene of Ilorin)

Since all the above elements do not exist, we regard the sets as empty or null sets. However, we cannot discuss empty set without mentioning infinite set because they are opposites.

Examples of infinite set are:

{All odd numbers}

{Animals that live in the forest}

P = { 5 x: x>2, x is even}

The elements of infinite sets are uncountable. But if members or elements of a set is countable, then the set is said to be a finite set e.g

{Prime numbers between 1 and 20}

{Number of lecturers in CAILS}

{Students of CAILS not older than 25years)

Sub Sets

Suppose a pot contains a fish, a goat meat and a cow meat. Now a boy is asked to choose one out of the three sets.

{Fish}, {Goat meat}, {Cow meat}, { }

{Fish, Goat meat}, {Fish Cow meat}

{Goat meat, Cow meat}

The original set is {Fish, Goat meat, Cow meat}

Therefore {Goat meat, Cow meat} C {Fish meat, Cow meat}

C: Proper sub set

C̲: Either proper or improper subset

e.g

1. Consider set A = {a, b, c, d, } and B = { a, b}

   BCA is {a, b} C̲ {, b, c, d}

2. Consider set β = { x: 1 $\leq$ x $\leq$ 12} x is a primer no

   i.e = {2, 3, 5, 7, 11)

   if λ = {2, 5, 11}

   = {1, 12}

Therefore λ C̲ β Lander is a subset of Beta

αC̲ β Alpha is not a subset of Beta

Equality of sets

Two sets $\propto$ and β are said to be equal if and only if $\alpha C \beta$ and $\beta C \alpha$. Suppose set α = {a, b, c} and set α ={b, c, a} then $\alpha = \beta$.

Note the arrangement of the members or elements of the set does not alter the set.

E.g. if G = {1, 2, 3, 4, 6} T = {4, 2, 1, 6}

GC̲T and TC̲G G=T

The Universal Set

The universal set can be described as the original given set which contains all the elements which can be used in a given problem. It is a larger set, since every set is a subset of a larger set which contains all elements to be discussed.

Universal set is symbolized as μ or ε e.g. Q if a die is tossed once, we are expected to have 1, 2, 3, 4, 5, or 6 as result. If there are no other expected results different from the six numbers above, then we say the universal set for this experiment is {1, 2, 3, 4, 5, 6}.

The universal set for tossing a coin is {H, T} H representing Head and T representing Tail.

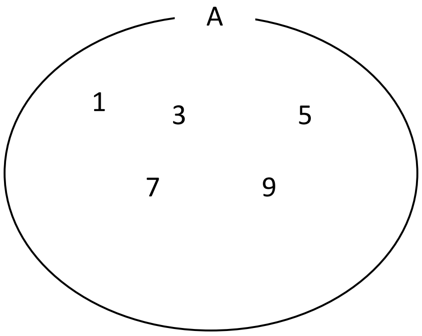
A singleton is a set with just one element. E.g. T = {a}.

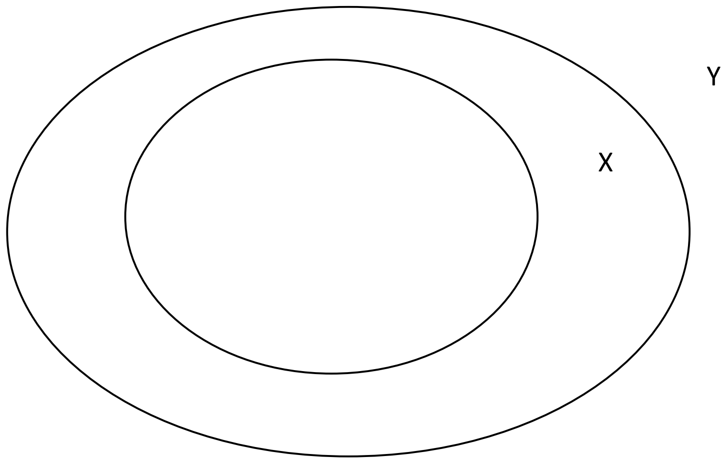
Venn Diagram

Venn Euler diagram is a sketched drawing used in showing the members of a set. Element within a boundary of a set falls inside the diagram.

e.g. A = {Odd numbers between 0 and 10}

A = {1, 3, 5, 7, 9}

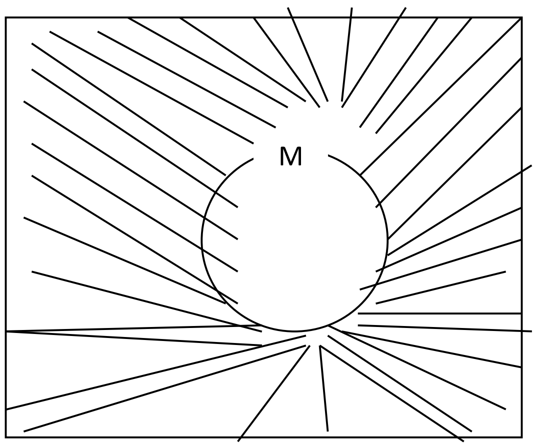




$x \subseteq Y$

Venn diagram are used to solve problems obtained from any data since it also involves numerical information.

Venn diagram cannot be discussed without mentioning universal set, because it is the container of other subsets e.g. M is a subset of a universal set μ i.e. $M \subseteq \mu$



Union and Intersection of Sets

The union of two sets makes the third set. This third set contains all the elements of the first two sets. The symbol U means union.

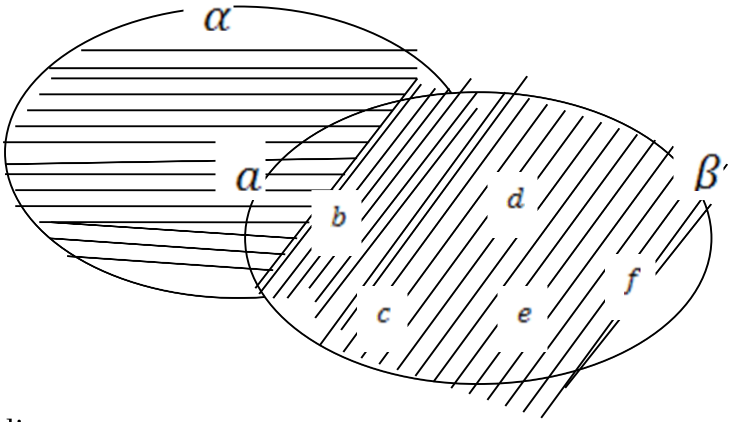
The intersection of two sets also makes the third set. The third set contains only those elements which are in both of the first two sets. The symbol ∩ means intersection.

For example, union and intersection could be illustrated on Venn Euler diagram as follows:

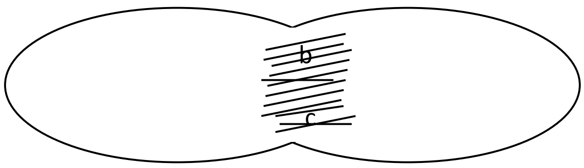
$\alpha \cup \beta$ = {a, b, c, d, e, f}

This implies

$\alpha \cap \beta$ = { b, c}



This implies



In (a) above, the shaded region represent the union of α and β whole in (b) the shaded region represent the intersection of α and β.

We can represent on a Venn diagram the union and intersection of the following pairs of sets.

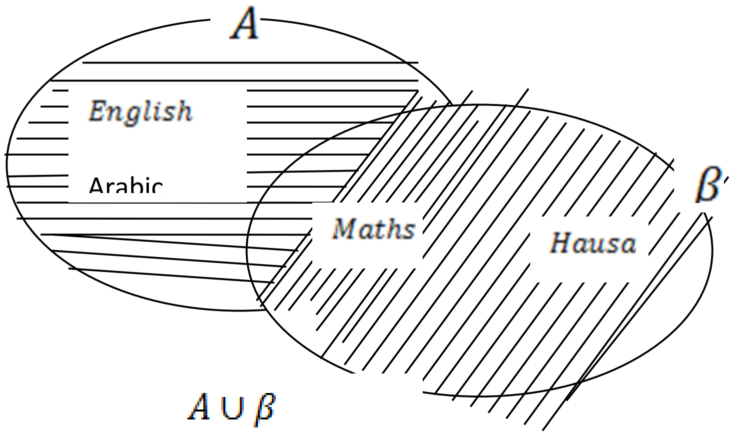
A = {English, Maths, Arabic}

B = {Maths, Hausa}

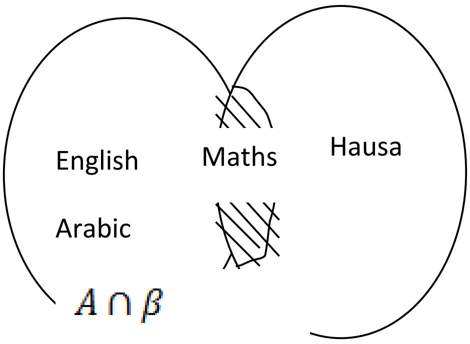
X = {1, 2, 3, 4} Y = {5, 6, 7, 8}

Solution:

$A \cup \beta =$ (English, Maths, Arabic, Hausa)



$A \cup \beta$



$x \cup Y$ = (1, 2, 3, 4, 5, 6, 7, 8) (ii)

$X \cap Y$

The above (ii) X ⋃Y disjoint because their interse

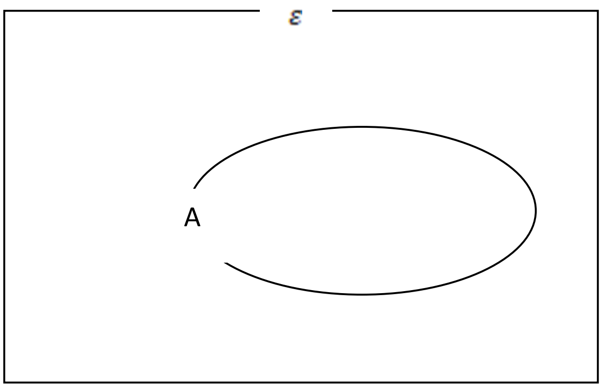
In a set language, we define AVB as A⋃B = {x: X∈A or X∈B}

And we define A⋂B as: A⋂B = {X∈A and X∈A}

<u>Complement of a Set</u>

The complement of a set A is the set of elements which are not in A, and it is written as A’.

Suppose A is a subject of the universal set ε, that is A<u>C</u>ε.



The set of all elements of ε above which are not in A is called A complement i.e. A^1 or A^c in a set language, it is written as:

$A^c = \{X: X \in \varepsilon, x \not\exists A\}$.

Sets Problems

Suppose = {1, 2, 3, 4, 5, 6} A = {1, 4}, B={2, 6}

Find (a) A^1 (b) B^1 (c) $\{AVB\}^1$ (d) $\{AnB\}^1$

Solution

$A^1 = \{2, 3, 5, 6\}$

$B^1 = \{1, 3, 4, 5\}$

Since {AVB} = {1, 2, 4, 6} ….. $\{AVB\}^1 = \{3, 5\}$

Since {AnB} = { }….. $\{AnB\}^1 = \{1, 2, 3, 4, 5, 6\}$

Given that x is an integer, list the numbers of $\{x{:}x<4\}$ n $\{X: \geq 2\}$

Solution

If XEZ

$\{x{:}x<4\} = \{\ldots.\ -3, -2, -1, 0, 1, 2, 3\}$

$\{x{:}x\geq -2\}$ {-2, -1, 0, 1, 2, 3}

$\{x{:}x<4\}$ n $\{x{:}x\geq -2\} = \{-2, -1, 0, 1, 2, 3\}$

Note that the above can be rewritten in short forms as $\{x: -2, \leq x < 4\}$ i.e. range of values.

Given that A, B and C are subsets of the universal set E, each of which is defined as follows:

$E = \{x: 2 \leq x < 12$, x is an integer}

$A = \{x: 3 < x < 6\}$

$B = \{x: (2 < x \leq 5)\ v\ (9 < x < 12)\}$

$C = \{x: 4 \leq x \leq 8\}$

List the members of sets E, A B and C

Find (i) (A∪B)∪C (ii) $A \cup (A \cap B)$ (iii) $A \cap (B \cup C)^1$

Solution

E = {2, 3, 4, 5, 6, 7, 8, 9, 10, 11}

A = {4, 5} B = {3, 4, 5, 10, 11}, C = {4, 5, 5, 7, 8}

(i) $A \cup B$ = { 3, 4, 5, 10, 11} $A \cup B \quad \cup C$ = {3, 4, 5, 6, 7, 8, 10 , 11}

(ii) $B \cap C$ = {4, 5} (A ∪) $(B \cap C)$ = {4,5}

(iii) B ∪ C = { 3, 4, 5, 6, 7, 8, 10, 11}, $(B \cup C)^1$ = {2, 9}, A∩ $(B \cup C)^1$ =Φ

Set Problems Involving Venn Diagrams

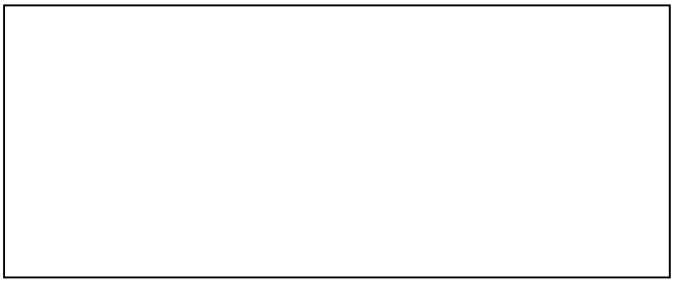
Venn diagrams can sometimes be used to student numerical information. in such cases it is also used to solve problem arising data.

Example (1)

50 Student were asked what they did last weekend. 16 said they went for sports, 14 said they watched television. if 7 said they did neither, how many did both.

Solution

Let Ɛ= (all students), S=(Sports students) and T=(Television watchers). it is required to find n (B∩T) =x. Let n(B∩T)=x. i.e.



Since n(T) =41, $n(S^1 nT)$ =41-X and $n(SUT)^1$=7

Similarly $n(SnT)^1$ = 16 –x and n $(SUT)^1$ = 7

The total for the regions must add up to the number of people in the universal set.

$$X + (16 - x) + (4 - x) + 7 = 50$$

$$64 - x = 50$$

$$x = 14$$

14 students went for sports and watched television. The 16 who went for sports includes the 14 who also watched television.

REVISION EXERCISES

If A = (3, 5, 6, 8, 9) and B = {2, 3, 4, 5} write down the sets A∪ B and $A \cap B$. Show A and B on a Venn diagram.

What is the union of (January, February, March) and (April, May, June)? What is their intersection? Represent the two sets on a Venn diagram.

Let U = {1, 2, 3, 4, ---------10)

A = (Odd members)

B = Numbers less than 7)

Write down the members of the following (a) AμB (b) A ∩ B (c) A∪ μ (d) A ∩ μ (e) μ ∩ B (f) μ ∪ B

Which of the following pairs of sets are disjoint?

{Even numbers}, (Odd numbers)

{Houses in Africa}, {Houses in teacher}

{Letters in pupils}, { Letters in teacher}

In a village all the people speak Hausa or English or both. if 97% speak Hausa and 64% speak English, what percentage speak both languages?

Out of 25 teachers, 16 are married and 15 are women. if 6 of the man married, how many of the women are not married?

In a class of 36 students, 29 do Mathematics and 20 do Arabic. If 5 students do neither, how many students do Arabic and not Mathematics?

In the following, state which set is finite or infinite,

Days of the week

The hairs on a human

All straight lines which passes through the coordinate axes

The set of herrings in the ocean

Find the solutions et in each of these

{X: 2x + 3 $<$ 7, XER}

{X: x2 -3x + 2 = 0, XER}

{X: X2 + 1 = 0, X is real}

State the universe in each of the following

When a coin is tossed

When two dice are tossed together

In a class of 50 students, 25 offer mathematics, 22 offer physics and 30 offer chemistry and all the students take at least one of these subjects. 10 offer physics and mathematics, 8 offer chemistry and mathematics, 16 offer, physics and chemistry.

Draw a Venn-Euler Diagram to illustrate the information

Find the number of students who offer all three subjects.

In a school of 530 students, 280 study Igbo, 210 study Hausa and 70 study no languages. How many study both Igbo and Hausa.

Write down the members of the given list which belong to the following sets

{Human beings}

{Living things}

{Liquids}

{Things normally found on a bus}

In each of the followings sets, pick out the member that is out of place. Give reasons.

{Lagos, Accra, Freetown, Cameroon}

{Bee, Airplane, Wasp, Fly}

{Table, Cow, Chair, Chicken, Dog}

{2, 4, 6, 7, 8}

{2+4}, {11-5}, {2x5}, {24:4}

Write down the next three members of the following sets

{3, 7, 11, 15 ….}

{2, 4, 8, 16 ….}

{7, 14, 21, 28 …}

{ 10, 21, 32, 42 …}

{1, 4, 9, 16 …}

If E = {earth, air, line, water} write down all the subjects of E

In preparing a time-table for a class of 38 students, the following facts were taken into consideration.

25 students take History

27 take French

28 take Agricultural Science

20 take both History and French

23 take French and Agricultural Science while

21 take both History and Agricultural Science

If 18 take all 3 subjects.

Express these in a Venn-Euler Diagram showing the number of students who offer the three subjects

How many students take only one subject?

How many students take at least two (2) subjects?

How many Offered none of the 3 subjects?

In a class of 58 students, 25 offer Mathematics, 22 offer Physics, 30 offer Chemistry and all the students take at least one of these subjects. 10 offer Physics and Mathematics, 8 offer Chemistry and Mathematics, 16 offer Physics and Chemistry.

Draw a Venn-Euler Diagram to illustrate the information

Find the number of students who offer all three subjects

Let μ represent the universal set of positive whole numbers. Let A, B, C be subset of μ defined as follows

A = {x: 0 $< x <$8, x is even}

B = {x: 2 $< x <$12, x is an integer}

C = {x: 1 $< x < 10$, x is a prime

Show that:

A ∪ (A∩C) = A

$A \cup B \cup C = A^1 \cap B^1 \cap C^1$

$A \cap (B \cap C) = (A \cap B) \cap (A \cap C)$

Given that A, B, and C are subsets of the universal setμ, each of which is defined as follows:

μ={x:2≤x<12,x is an interger}

A={x:3<x<6}

B={x:{2<≤5}Ú{9<x<12}

C={x:4≤x≤8}

List the members of sets U,A,B,C]

Find (i) (AUB)UC

(ii) AU(B∩C)

(iii)$A \cap (BUC)^1$

(a) Given the universal setμ={1,2,3,4,5} and P={1,2,4},

(a) ={2,4,5},find $p^1 \cap Q$

(b) if μ is the universal set consisting of all positive integers and P,Q are subsets such that:

P={x: x is a prime number}

Q = {x: x is a even number}

R = {x:7<x <20}

List the elements of

(i) P∩R (ii) Q^1∩R (iii) P^1∩(Q^1∩R)

Assuming that X and Y are not disjoint sets, indicate the following on Venn-Euler diagrams.

X^1 (b) X^1∩Y^1 (c) $(XUY)^1$ (d) X^1∩Y (e) (X∩Y)1

s if the universal set μ= is given by μ={a, b, c, d ,e, f, g, h} and the sets A, B and C are defined as follows

A= {a, b, c}, B= {c, d, e, g} and C= {b, c, d}

Find (i) AU (BUC) (ii) (A∩B) (iii) $A^1U^1B^1$ what do you notice about (ii) and (iii) (iv) $(A^1)^1$.

if μ ={x:0<x≤12, x is an integer}

T= {x: x is divisible by 2, Ɛμ}

S= {x: x is a prime x Ɛμ}

Find (i) TUS (ii) T∩S (iii) T^1 U S (iv) T^1 ∩S^1

Deduce that $(T \cap S)^1 = T^1 \cup S^1$

Given that the universal set μ= {a, b, c, d, e, f, g, h, I, j, k } and A={a, d, e, I, k,} B= {f, g , I, j, k } using the above sets, show that $(AU B)^1 = A^1 \cap B^1$.

The universal set μis the set of all integers. P,Q and R are subsets of U defined as follows:

P = {…, -5, -3, -1,3 ……}

Q = {x: -1<x<8, x is an integer}

R = {x: -3 < x 3, ∞ is an integer}

write down the set P^1, where P^1 is the complement of P with respect to μ.

write in set form P ∩Q, and List members of R show that P U (Q ∩R) = (PUQ) ∩ (P UR)

Resolve the following into partial fractions.

(a) $\frac{x+1}{(x-1)(x-2)}$ (b) $\frac{3x+8}{(x-2)(x+4)}$ (c) $\frac{1+x}{(1-x)(2+x)}$ (d) $\frac{2x+4}{(x-1)(x+3}$

REFERENCES

Further Mathematics: A Millennium Text by E. Egbe, .G.A. Odili O.O. Ugebor.

New general Mathematics for west African SS1 with other fabricated examples and questions.